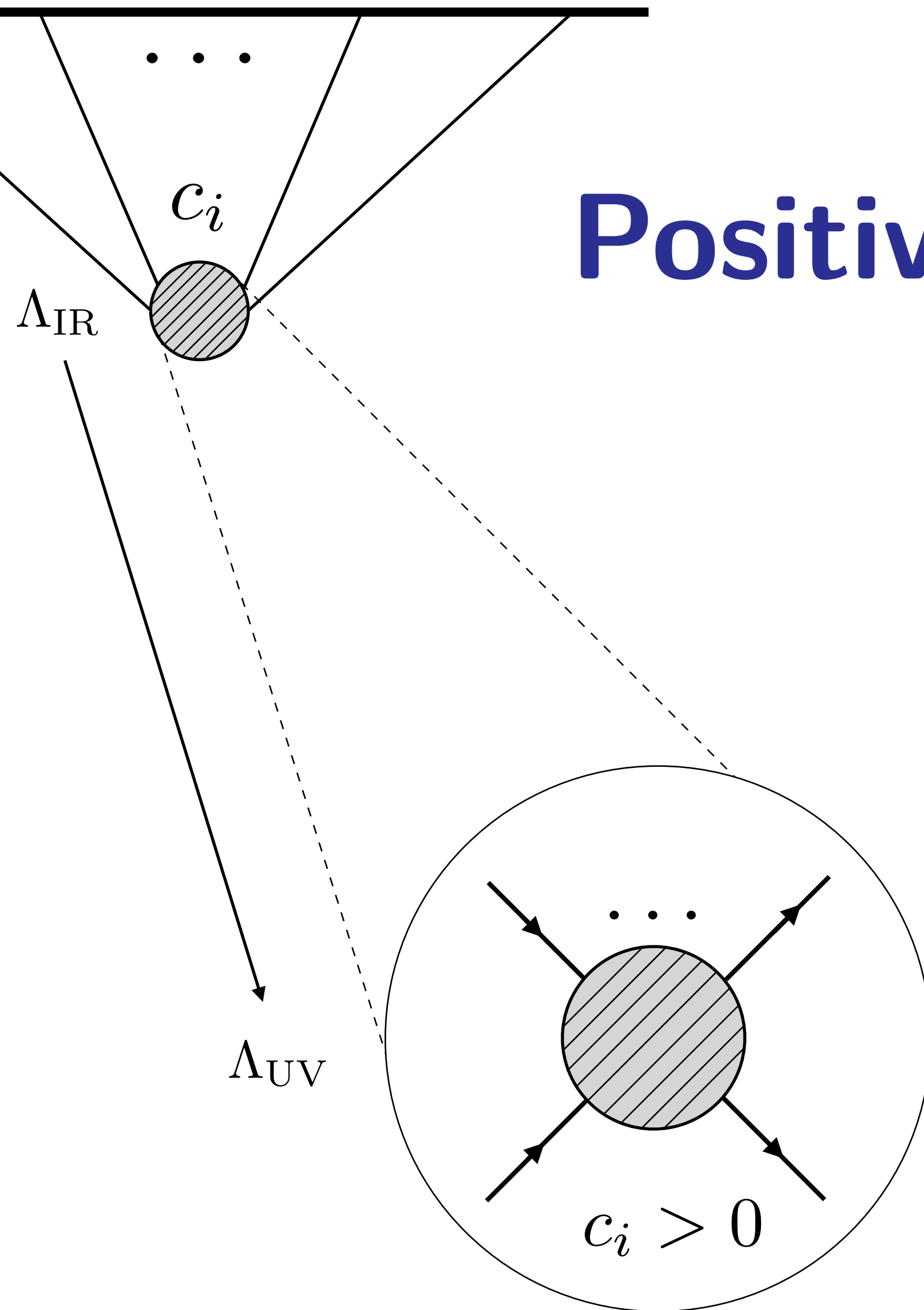


Positivity constraints on the EFT of inflation

Rafael Bravo

Pontificia Universidad Católica de Chile

Work in progress with S. Cespedes (UCHile), N. Parra (KIT) and S. Riquelme (USS)



Motivation:

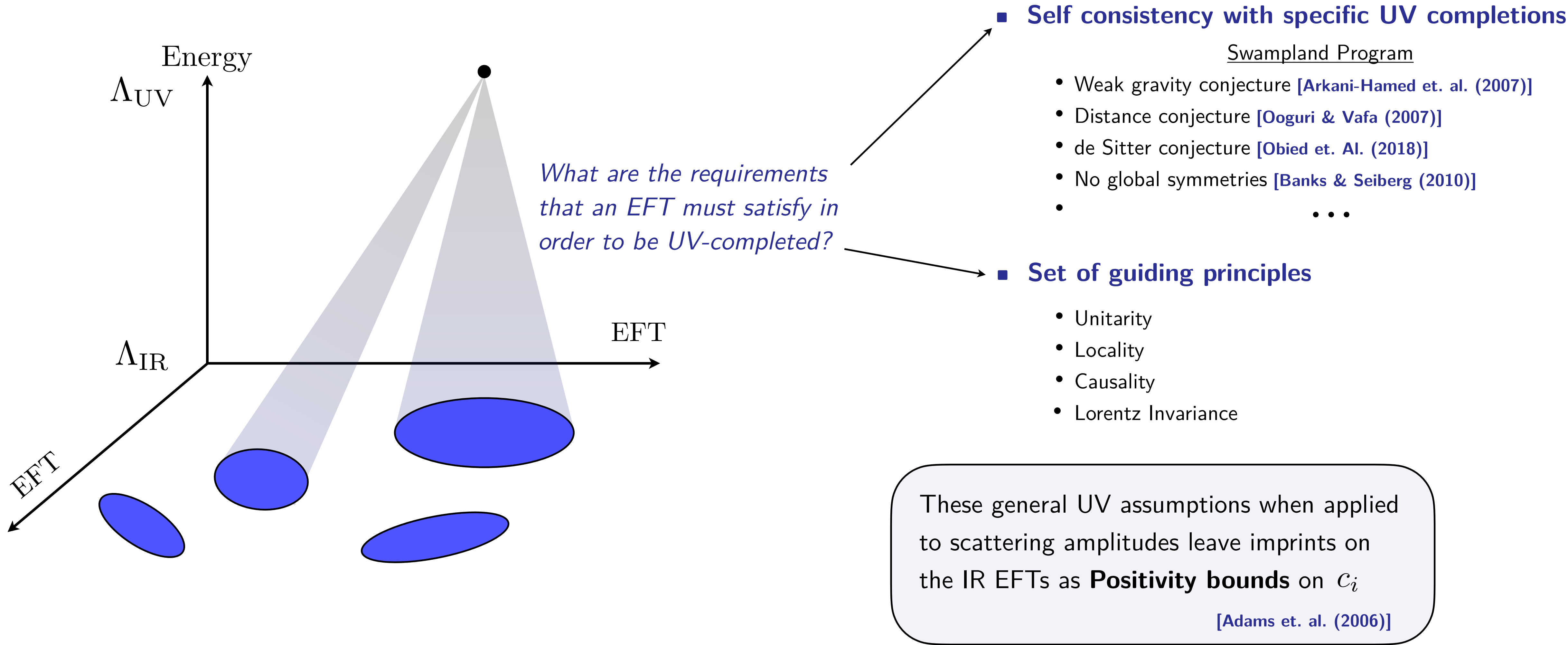
UV constraints on IR Physics

UV constraints on IR Physics

$$\mathcal{L}_{\text{EFT}}[\phi] = \mathcal{L}_0[\phi] + \sum_i c_i \frac{\mathcal{O}_i[\phi]}{\Lambda^{d_i-4}}$$

Q: Can every EFT be UV completed ?

A: No! Some low-energy EFT do not admit well defined UV completions



Positivity Bounds: a concrete example

- Low energy EFT of a massless shift symmetric scalar

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{c_1}{\Lambda^4}(\partial_\mu\pi\partial^\mu\pi)^2 \quad \pi \rightarrow \pi + c, \quad \pi \rightarrow -\pi$$

Expanding around a constant background gradient background $\partial_\mu\pi_0 = C_\mu$ $\pi = \pi_0 + \delta\pi$

$$k^\mu k_\mu + 4\frac{c_1}{\Lambda^4}(C \cdot k)^2 = 0 \quad (C \cdot k)^2 > 0 \quad \text{Subluminality} \implies c_1 > 0$$

Causality $\implies$ positivity

- If we were started from the UV theory of a broken U(1)

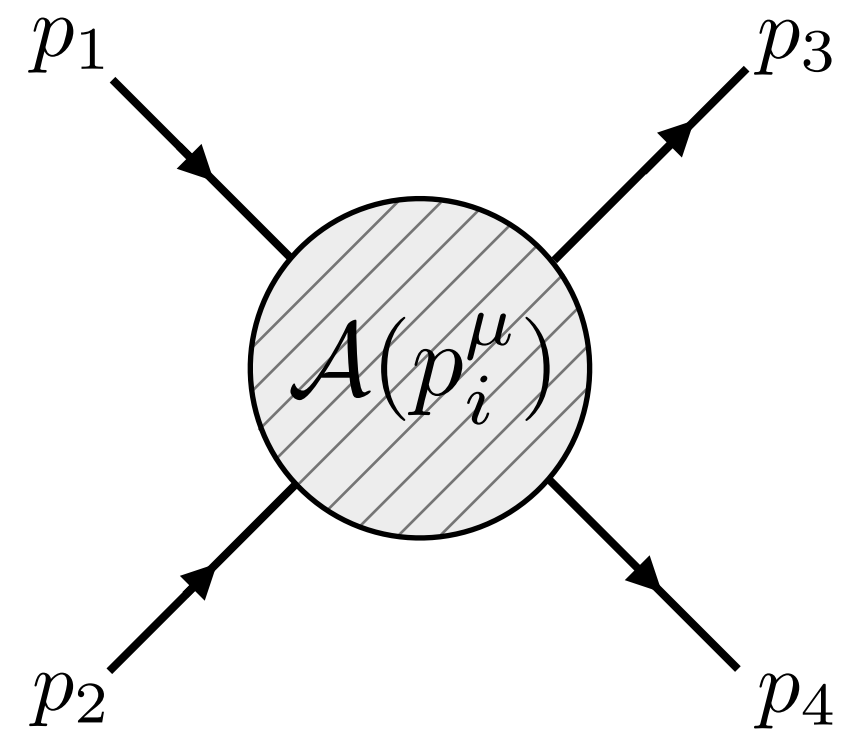
$$\mathcal{L} = -\frac{1}{2}(1+h)(\partial\pi)^2 - \frac{1}{2}(\partial h)^2 - M^2 h^2 - \frac{\lambda}{4}(h^2 + 2vh)^2$$

Integrating out h at tree-level

$$\mathcal{L}_{\text{EFT}}[\pi] = -\frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{\lambda}{M^4}(\partial_\mu\pi\partial^\mu\pi)^2 \quad \lambda > 0$$

Positivity Bounds from scattering amplitudes in flat space

$2 \rightarrow 2$ scattering for identical massive scalars of mass m



$$s + t + u = 4m^2$$

Properties

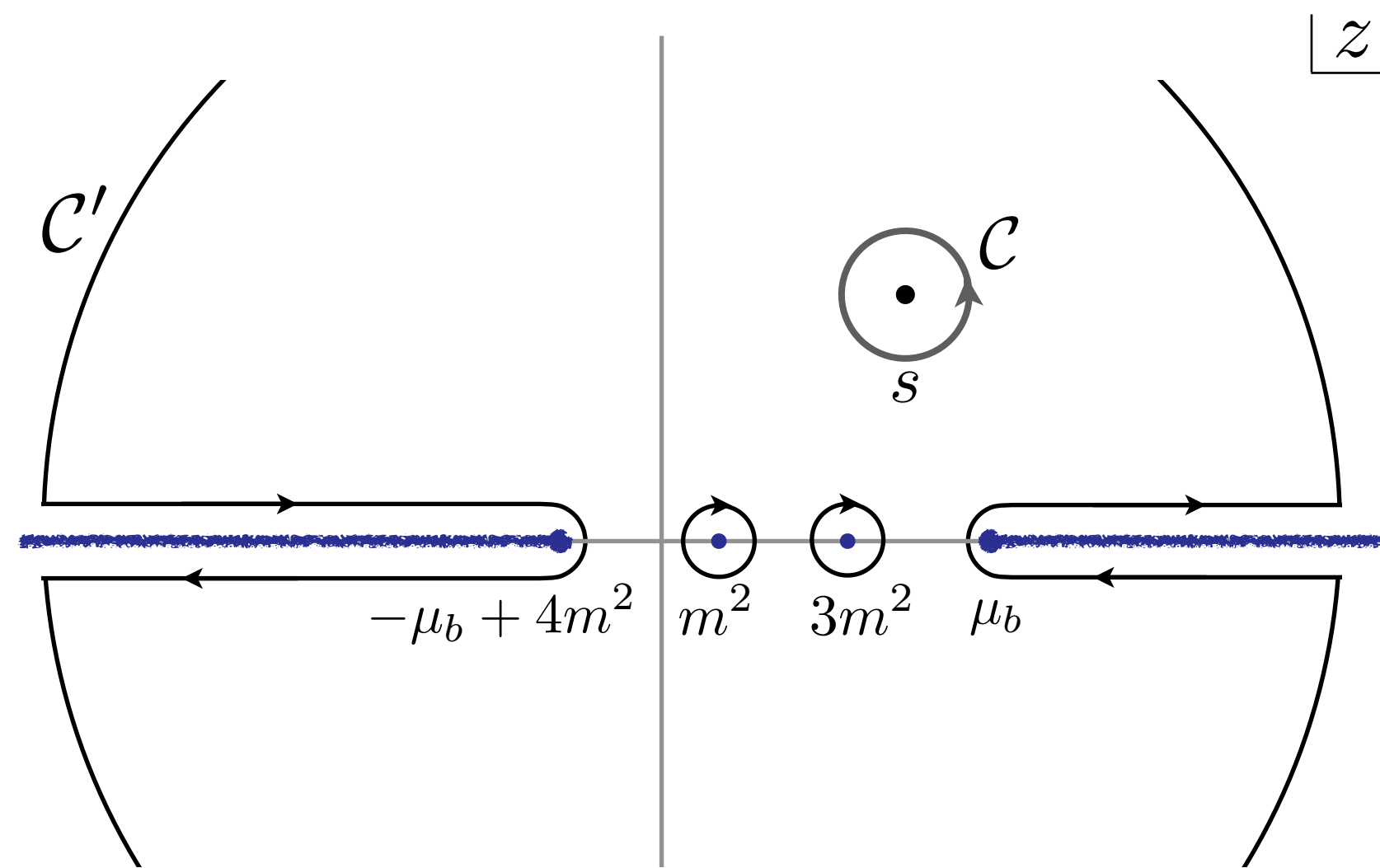
- Unitarity: $S^\dagger S = 1 \implies \text{Im } \mathcal{A}(s, 0) \propto s \sigma_{\text{tot}}(s) \geq 0$
- Locality: $|\mathcal{A}| \leq C s \log^2 s$
- Causality: $\mathcal{A}(s, t)$ is analytic
- Lorentz invariance $s \leftrightarrow -s$

Positivity Bounds

$$\left. \frac{\partial^{2n} \mathcal{A}(s)}{\partial s^{2n}} \right|_{s=0} \geq 0$$

[Adams et. Al (2006)]

[Arkani-Hamed, Huang, Huang (2020)]



$$\mathcal{A}(s, t) = \frac{1}{2\pi i} \oint_{\mathcal{C}} dz \frac{\mathcal{A}(z, t)}{z - s}$$

We can expand the amplitude in the forward limit $t \rightarrow 0$ as

$$\mathcal{A}(s, 0) = \mathcal{A}(0, 0) + s \mathcal{A}'(s, 0) + \frac{s^2}{\pi} \int_{4m^2}^{\infty} \frac{\text{Im} \mathcal{A}(z, 0)}{z^2(z - s)} dz + \dots$$

$$\mathcal{A}''(0, 0) = \frac{2}{\pi} \int_{4m^2}^{\infty} dz \frac{\text{Im} \mathcal{A}(z, 0)}{z^3} = \frac{2}{\pi} \int_{4m^2}^{\infty} dz \frac{\sigma_{\text{tot}}(z)}{z^2} \geq 0$$

For the theory

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \pi \partial^\mu \pi + \frac{c_1}{\Lambda^4} (\partial_\mu \pi \partial^\mu \pi)^2$$

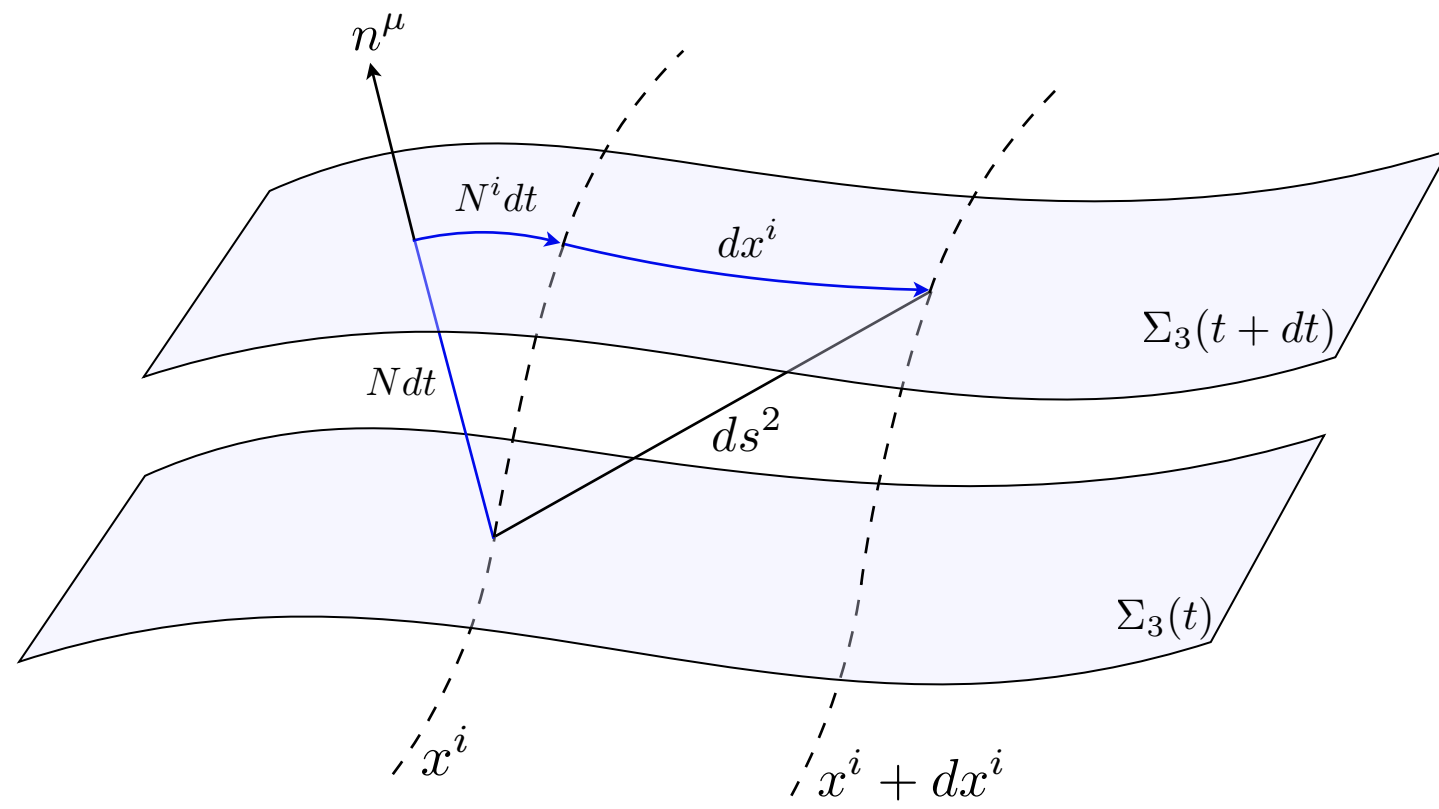
The corresponding 2-2 scattering amplitude

$$\mathcal{A}(s, 0) = 2 \frac{c_1}{\Lambda^4} s^2$$

The positivity bound implies $c_1 \geq 0$

Analytic structure in the forward limit $\mathcal{A}(s, t = 0)$

The EFT of inflation



Unitary gauge action

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + M_{\text{Pl}}^2 \dot{H} g^{00} - M_{\text{Pl}}^2 (3H^2 + \dot{H}) + \frac{M_2^4(t)}{2!} (\delta g^{00})^2 + \frac{M_3^4(t)}{3!} (\delta g^{00})^3 \right. \\ \left. - \frac{\bar{M}_1^3(t)}{2} (\delta g^{00}) \delta K^\mu{}_\mu - \frac{\bar{M}_2^2(t)}{2} (\delta K^\mu{}_\mu)^2 - \frac{\bar{M}_3^2(t)}{2} \delta K^\mu{}_\nu \delta K^\nu{}_\mu + \dots \right]$$

[Cheung et. al (2007)]

$t \rightarrow t + \pi(t, x)$ decoupling limit Stückelberg action

$$S_\pi = \int d^4x \left[M_{\text{Pl}}^2 \dot{H} (\partial_\mu \pi)^2 + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{1}{a^2} (\partial_i \pi)^2 \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 - \frac{\bar{M}^2}{2} \frac{1}{a^4} (\partial_i^2 \pi)^2 + \dots \right]$$

Can we say something about the signs of $M_i, \bar{M}_i$?

[Baumann et. al. (2015)]

[Grall & S. Melville (2021)]

[Creminelli et. al. (2023)]

[Freytsis et. al. (2023)]

[Green et. al. (2023)]

[Serra & Trombetta (2024)]

[Creminelli et. al. (2024)]

[Gordon Lee & S. Melville (2025)]

■ ~~Flat space~~ $\Rightarrow$ quasi dS

■ ~~Scattering amplitudes~~ $\Rightarrow$ equal-time correlation functions

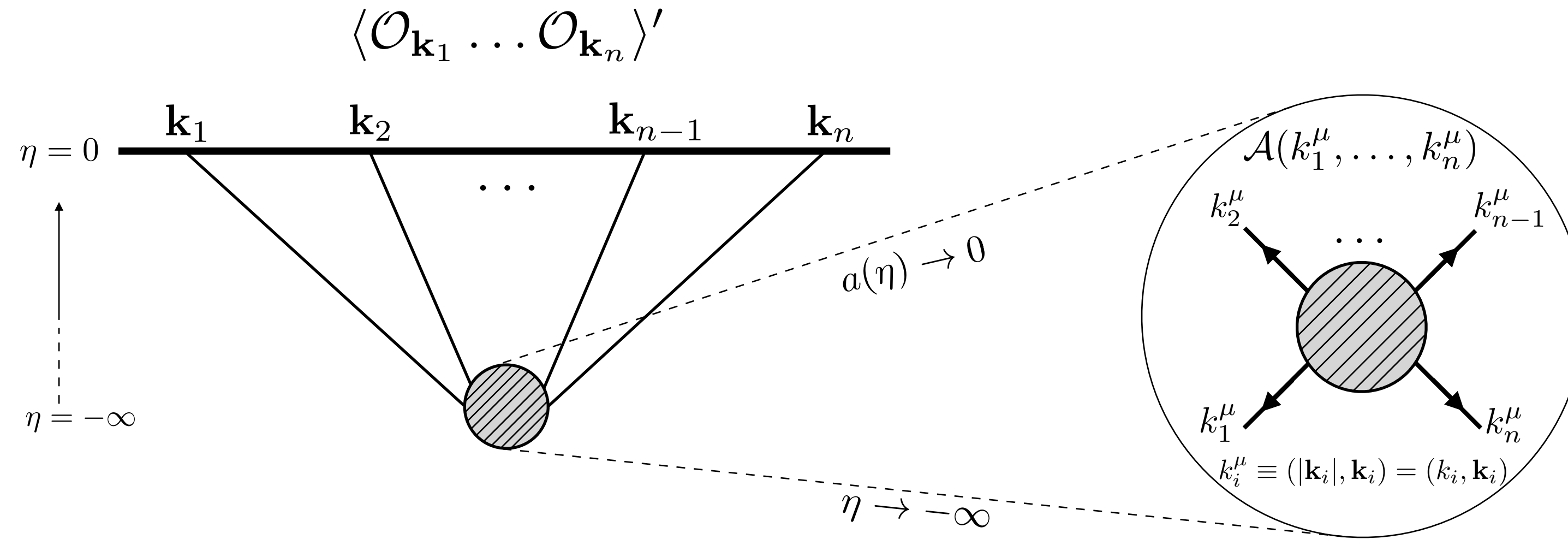
■ ~~Lorentz invariance~~ $\Rightarrow$ spontaneously broken, bonus: no energy conservation

Let's try with a spherical cow

Lorentz breaking amplitudes
and
Amplitudes from dS correlators

The amplitude limit of cosmological correlators

Cosmological correlators contain flat scattering amplitudes!



$$\lim_{k_T \rightarrow 0} \langle \mathcal{O}_{\mathbf{k}_1} \dots \mathcal{O}_{\mathbf{k}_n} \rangle' = \frac{(-1)^n H^{p+n-1} \Gamma(p)}{2^{n-1} (\prod_{a=1}^n k_a^2)} \times \frac{\text{Re} (i^{1+n+p} \mathcal{A}(k_1^\mu, \dots, k_n^\mu))}{k_T^p}$$

Annotations:

- $k_T = \sum_a k_a$ (Total energy)
- $p = 1 + \sum_i (d_i - 4)$ (Mass dimension of the operator)
- k_T^p ("Energy" pole)

[Maldacena & Pimentel (2011)]
 [Arkani-Hamed & Maldacena (2015)]
 [Arkani-Hamed, Benincasa & Postnikov (2017)]
 [Pajer (2020)]

■ Scattering Amplitudes

$$\mathcal{A} \propto \int_{-\infty}^{\infty} dt \prod_a e^{e E_a t} = 2\pi \delta(E_T)$$

■ Correlation functions

$$\langle \mathcal{O}_{\vec{k}_1} \dots \mathcal{O}_{\vec{k}_n} \rangle' \propto \int_{-\infty}^0 d\eta \eta^{p-1} \prod_a e^{i k_a \eta} = \frac{1}{k_T^p}$$

At early times

$$a(\eta) \rightarrow 0$$

$$\varphi_k(\eta) \sim \frac{1}{\sqrt{2k}} e^{-ik\eta}$$

The mode functions are the same that in Minkowski

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