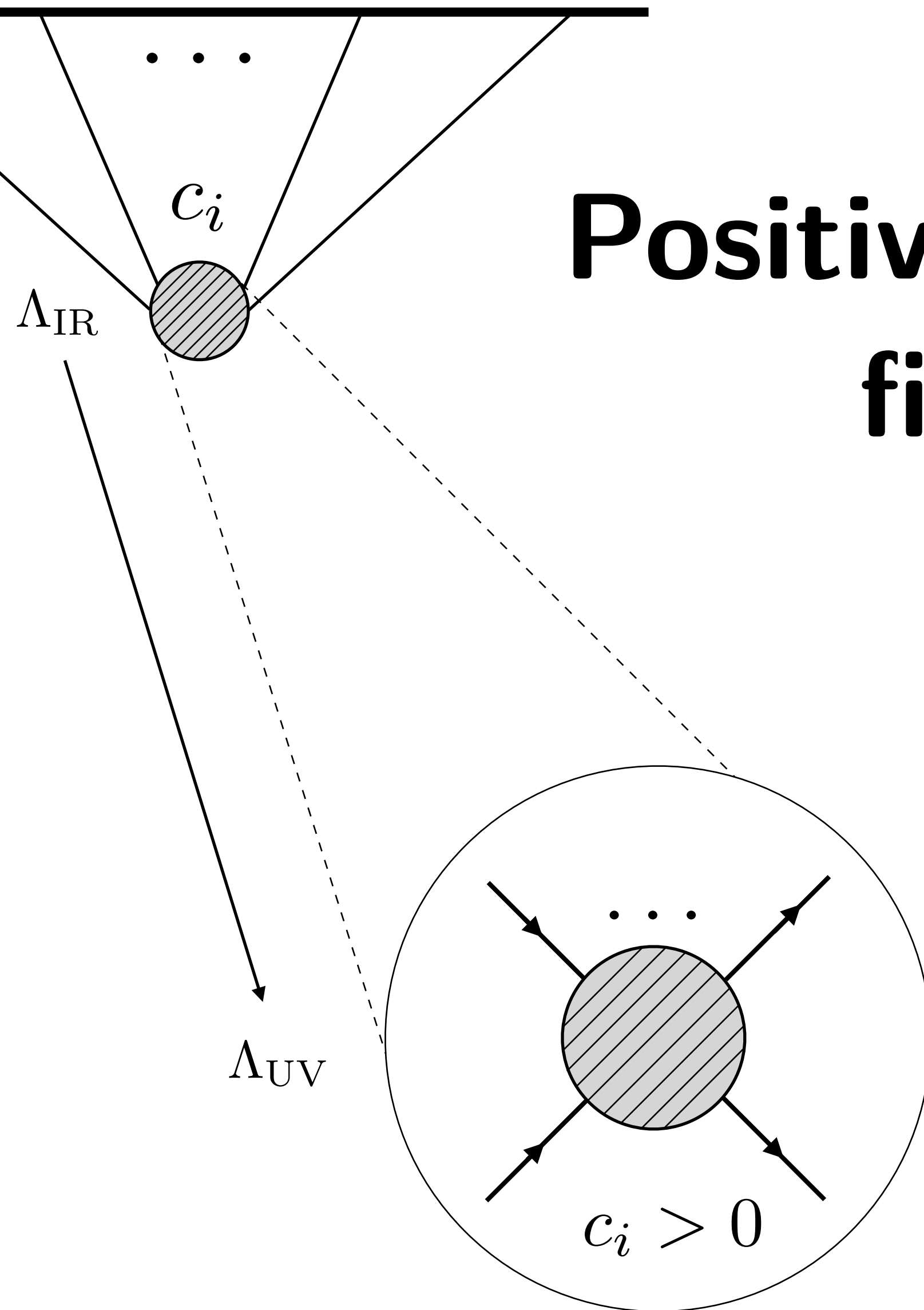


Positivity bounds on the effective field theory of inflation

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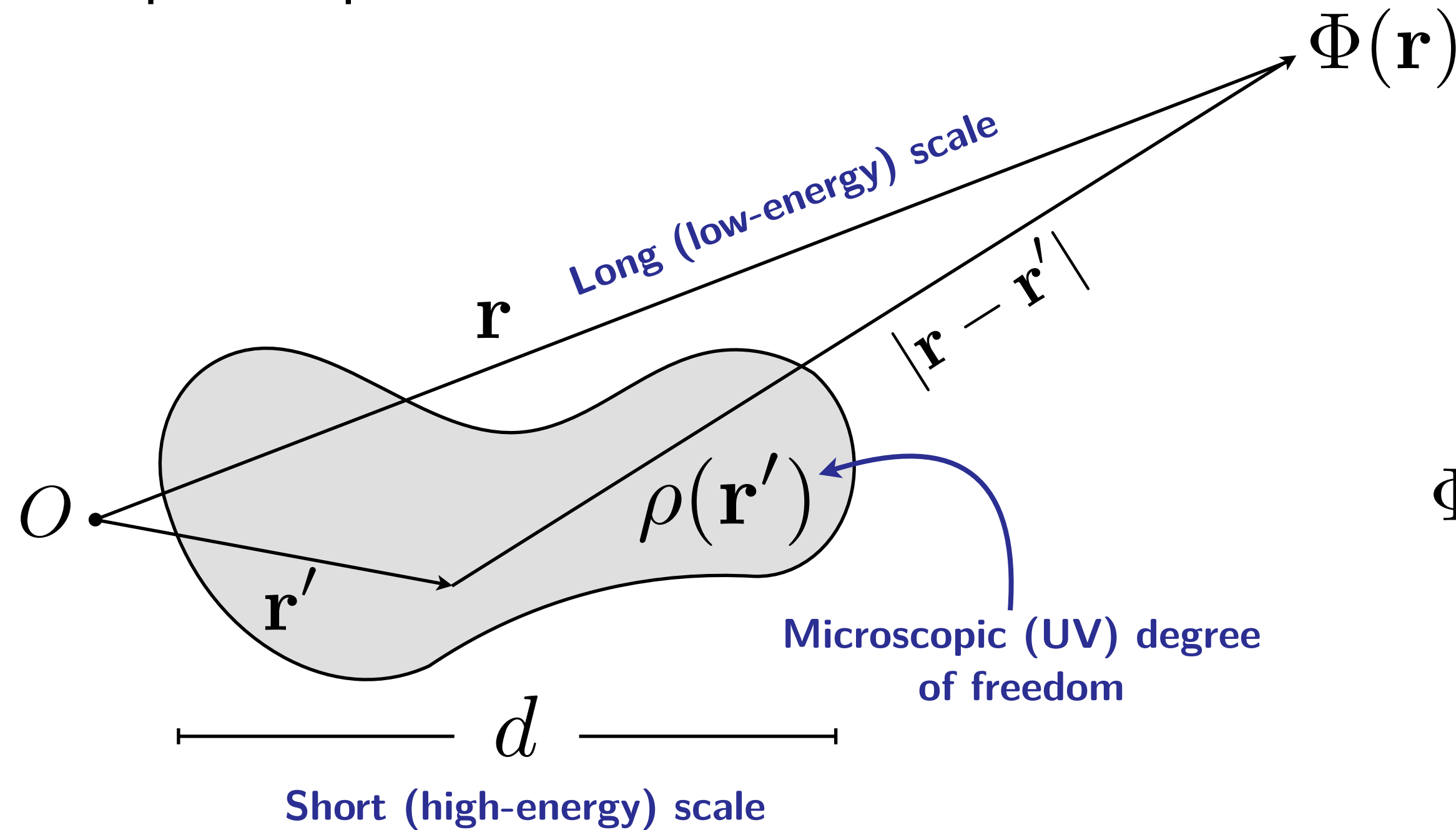


Motivation:

UV constraints on IR Physics

Warm up: why effective theories?

■ Multipole expansion in electrostatics



At large distances, $r \gg d$: Validity region of the low-energy (IR) EFT expansion

$$\Phi(\mathbf{r}) = \frac{q}{4\pi\epsilon_0 r} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} Q_{\ell m} \left(\frac{d}{r} \right)^{\ell} Y_{\ell m}(\theta, \varphi)$$

Annotations for the multipole expansion:

- Dimensionless coefficients**: points to $Q_{\ell m}$
- Local operators**: points to $Y_{\ell m}(\theta, \varphi)$
- Small (IR/UV) expansion parameter (cutoff)**: points to $\left(\frac{d}{r} \right)^{\ell}$
- Infinite expansion**: points to the summation over ℓ

Full electrostatic potential **UV theory**

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int d^3\mathbf{r}' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

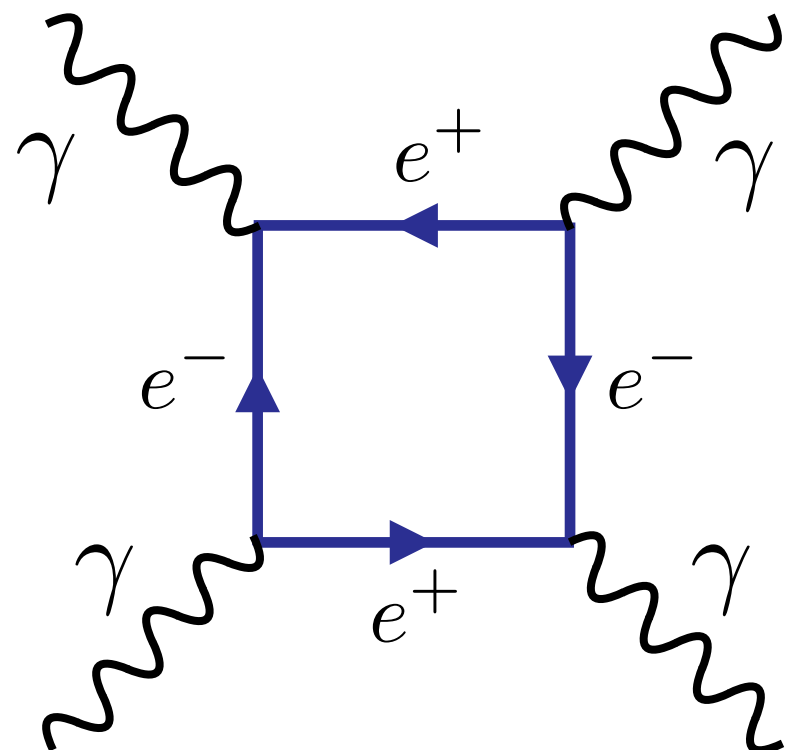
Multipole moments

$$Q_{\ell m} \sim \int_V d^3\mathbf{r}' \rho(\mathbf{r}') r'^{\ell} Y_{\ell m}^*(\theta', \varphi')$$

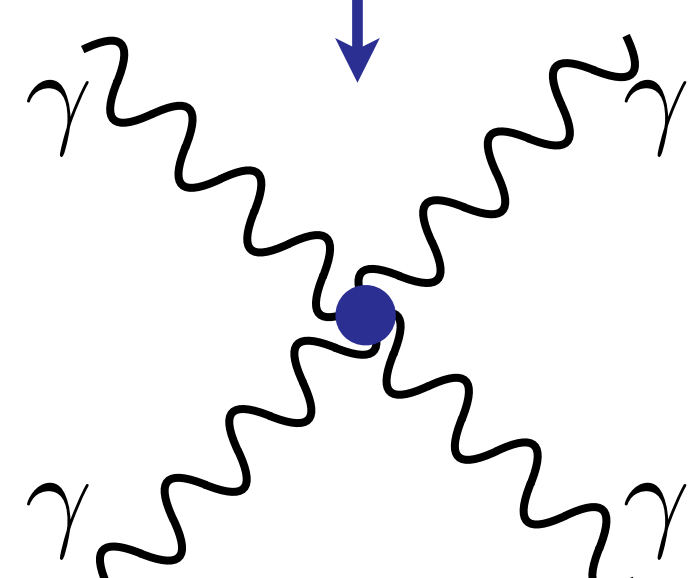
Dimensionless coefficients encode information about the microscopic (UV) degree of freedom

EFTs in quantum field theory

- Low-energy photon-photon scattering $\gamma\gamma \rightarrow \gamma\gamma$



$E \ll m_e$



$$\mathcal{L}_{\text{QED}}[A_\mu, \psi, \bar{\psi}] = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{D} - m_e)\psi$$

$$\mathcal{A}_{\text{UV}} = \alpha^2 \left[c_2(\{\lambda_i\}) \frac{s^2 + t^2 + u^2}{m_e^4} + c_3(\{\lambda_i\}) \frac{stu}{m_e^6} + \dots \right]$$

$$e^{i S_{\text{eff}}[A_\mu]} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{i S_{\text{QED}}[A, \psi, \bar{\psi}]}$$

$$\mathcal{L}_{\text{EH}}[A_\mu] = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{2\alpha^2}{45m_e^4} \left[1 (F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4} (F_{\mu\nu}\tilde{F}^{\mu\nu})^2 \right] + \mathcal{O}(1/m_e^8)$$

UV cutoff scale



U(1) invariant operators



Dimensionless Wilson coefficients



$$\mathcal{A}_{\text{IR}} = \frac{\alpha^2}{m_e^4} \left[a(\{\lambda_i\}) s^2 + b(\{\lambda_i\}) (t^2 + u^2) \right]$$

Wilsonian EFT approach

- Top-bottom: the **UV** theory is **known**

$$\mathcal{L}_{\text{UV}}[\ell, H] = \mathcal{L}_{\text{light}}[\ell; m_\ell] + \mathcal{L}_{\text{heavy}}[H; M] + \mathcal{L}_{\text{int}}[\ell, H], \quad m_\ell \ll M$$

→ Integrate out heavy fields $e^{iS_{\text{eff}}[\ell]} = \int \mathcal{D}H e^{iS[\ell, H]}$

$\Rightarrow$

$$\mathcal{L}_{\text{EFT}}[\ell] = \mathcal{L}_{\text{light}}^{(\text{ren})}[\ell] + \sum_i \frac{c_i(\mu)}{M^{d_i-4}} \mathcal{O}_i(\ell)$$

→ Expand in local operators built from ℓ

- Bottom-up: the **UV** theory is **unknown**

→ Identify the relevant degrees of freedom ϕ and scales Λ

→ Determine the relevant (broken) symmetries

→ Write all operators compatible with the symmetries

→ Compute observables

→ Measure parameters, fix the Wilson coefficients c_i

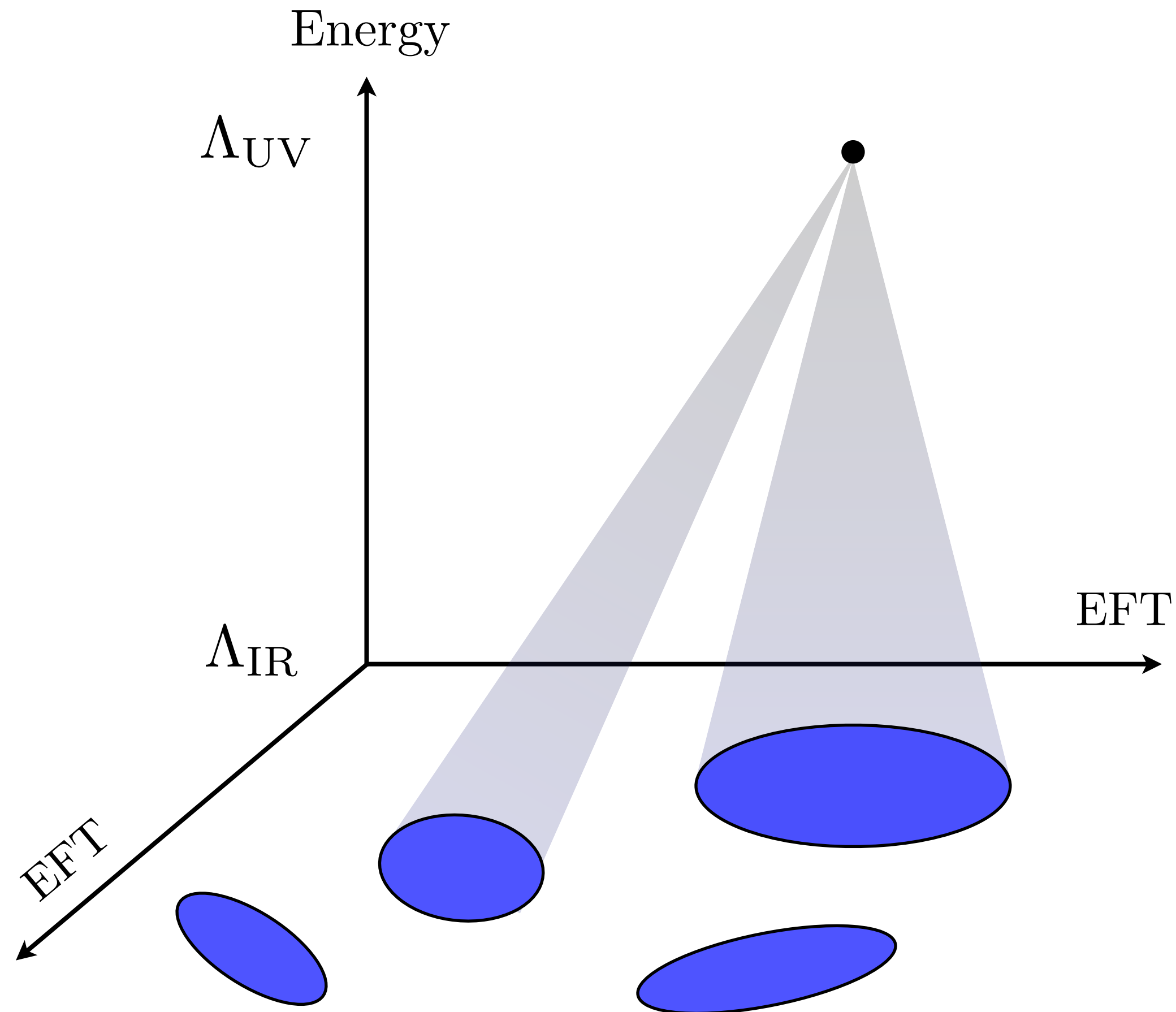
$\Rightarrow$

$$\mathcal{L}_{\text{EFT}}[\phi] = \mathcal{L}_0[\phi] + \sum_i c_i \frac{\mathcal{O}_i[\phi]}{\Lambda^{d_i-4}}$$

The Wilson coefficients
know about the UV theory

UV constraints on IR Physics

Can every EFT be UV completed into a full theory? → No! Some low-energy EFT do not admit well defined UV completions



$$\mathcal{L}_{\text{EFT}}[\phi] = \mathcal{L}_0[\phi] + \sum_i c_i \frac{\mathcal{O}_i[\phi]}{\Lambda^{d_i-4}}$$

What are the requirements that an EFT must satisfy in order to be UV-completed?

■ Self consistency with specific UV completions

String theory swampland program

- Weak gravity conjecture [Arkani-Hamed et. al. (2007)]
- Distance conjecture [Ooguri & Vafa (2007)]
- de Sitter conjecture [Obied et. Al. (2018)]
- No global symmetries [Banks & Seiberg (2010)]
- ...

■ Set of guiding principles

- Unitarity
- Locality
- Causality
- Lorentz Invariance

These general UV assumptions when applied to scattering amplitudes leave imprints on IR EFTs as **Positivity bounds** on $\mathcal{C}_i$

[Adams et. al. (2006)]

Positivity Bounds: a concrete example

- Low energy EFT of a massless shift symmetric scalar

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{c_1}{\Lambda^4}(\partial_\mu\pi\partial^\mu\pi)^2 \quad \pi \rightarrow \pi + c, \quad \pi \rightarrow -\pi$$

Expanding around a constant background gradient background $\partial_\mu\pi_0 = C_\mu$ $\pi = \pi_0 + \delta\pi$

$$k^\mu k_\mu + 4\frac{c_1}{\Lambda^4}(C \cdot k)^2 = 0 \quad (C \cdot k)^2 > 0 \quad \text{Subluminality} \implies c_1 > 0$$

Causality $\implies$ positivity

- If we were started from the UV theory of a broken U(1)

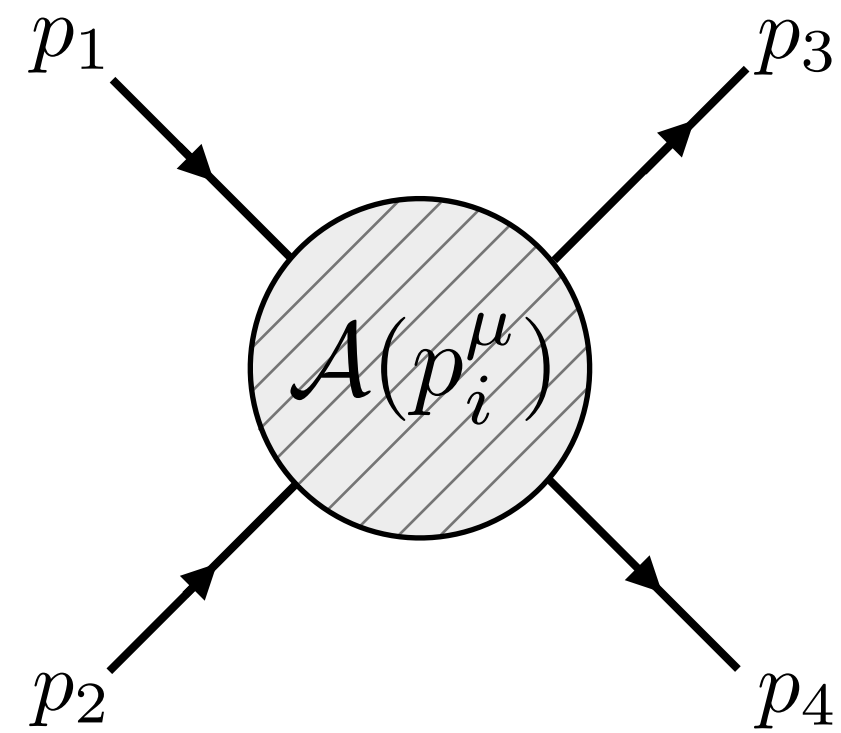
$$\mathcal{L} = -\frac{1}{2}(1+h)(\partial\pi)^2 - \frac{1}{2}(\partial h)^2 - M^2 h^2 - \frac{\lambda}{4}(h^2 + 2vh)^2$$

Integrating out h at tree-level

$$\mathcal{L}_{\text{EFT}}[\pi] = -\frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{\lambda}{M^4}(\partial_\mu\pi\partial^\mu\pi)^2 \quad \lambda > 0$$

Positivity Bounds from scattering amplitudes in flat space

$2 \rightarrow 2$ scattering for identical massive scalars of mass m



$$s + t + u = 4m^2$$

Properties

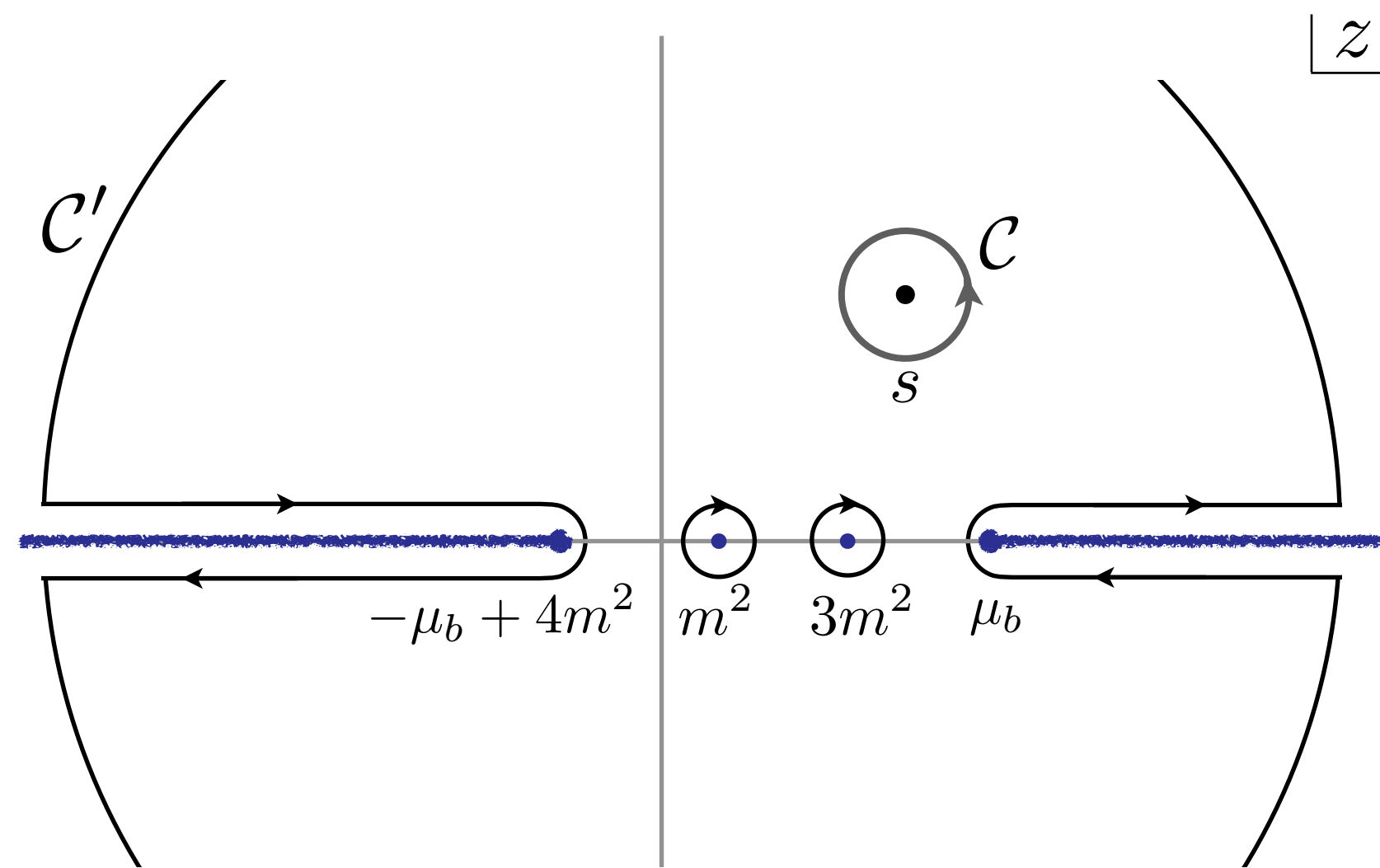
- Unitarity: $S^\dagger S = 1 \implies \text{Im } \mathcal{A}(s, 0) \propto s \sigma_{\text{tot}}(s) \geq 0$
- Locality: $|\mathcal{A}| \leq C s \log^2 s$
- Causality: $\mathcal{A}(s, t)$ is analytic
- Lorentz invariance $s \leftrightarrow -s$

Positivity Bounds

$$\left. \frac{\partial^{2n} \mathcal{A}(s)}{\partial s^{2n}} \right|_{s=0} \geq 0$$

[Adams et. Al (2006)]

[Arkani-Hamed, Huang, Huang (2020)]



$$\mathcal{A}(s, t) = \frac{1}{2\pi i} \oint_C dz \frac{\mathcal{A}(z, t)}{z - s}$$

We can expand the amplitude in the forward limit $t \rightarrow 0$ as

$$\mathcal{A}(s, 0) = \mathcal{A}(0, 0) + s \mathcal{A}'(s, 0) + \frac{s^2}{\pi} \int_{4m^2}^{\infty} \frac{\text{Im} \mathcal{A}(z, 0)}{z^2(z - s)} dz + \dots$$

$$\mathcal{A}''(0, 0) = \frac{2}{\pi} \int_{4m^2}^{\infty} dz \frac{\text{Im} \mathcal{A}(z, 0)}{z^3} = \frac{2}{\pi} \int_{4m^2}^{\infty} dz \frac{\sigma_{\text{tot}}(z)}{z^2} \geq 0$$

For the theory

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \pi \partial^\mu \pi + \frac{c_1}{\Lambda^4} (\partial_\mu \pi \partial^\mu \pi)^2$$

The corresponding 2-2 scattering amplitude

$$\mathcal{A}(s, 0) = 2 \frac{c_1}{\Lambda^4} s^2$$

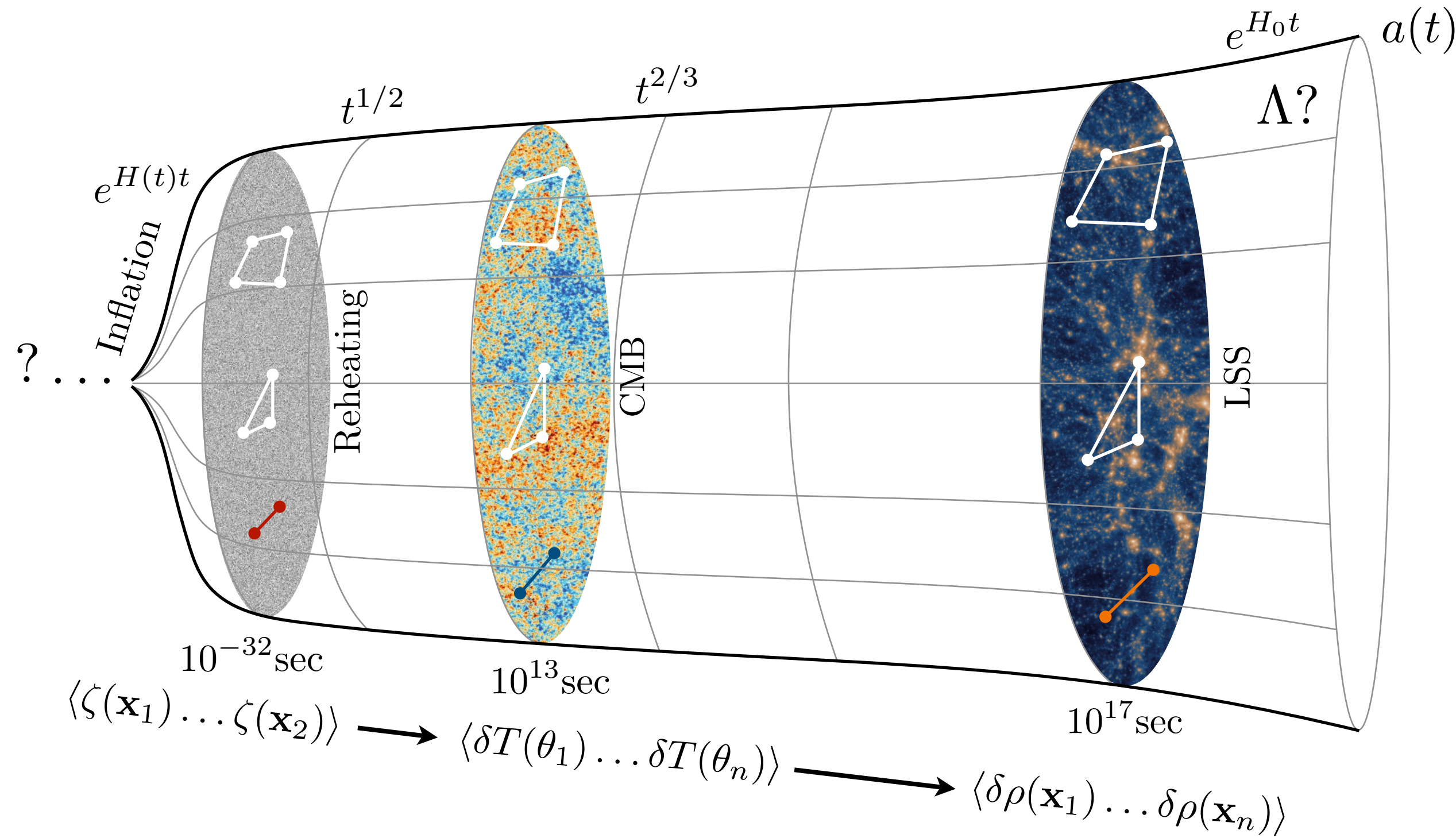
The positivity bound implies $c_1 \geq 0$

Analytic structure in the forward limit $\mathcal{A}(s, t = 0)$

EFTs in cosmology

EFTs in cosmological setups

Cosmology aims to build a **consistent history** of the spatial correlations of **perturbations** measured across large scales



Time dependent background matter fields *spontaneously* break time diffeomorphism invariance setting a preferred time direction.

$$t \not\rightarrow t + \xi^0(t, \mathbf{x})$$

Global version $\Rightarrow$ isometry generated by a time-like ξ^μ

The *vacuum* is the background (unperturbed) spacetime

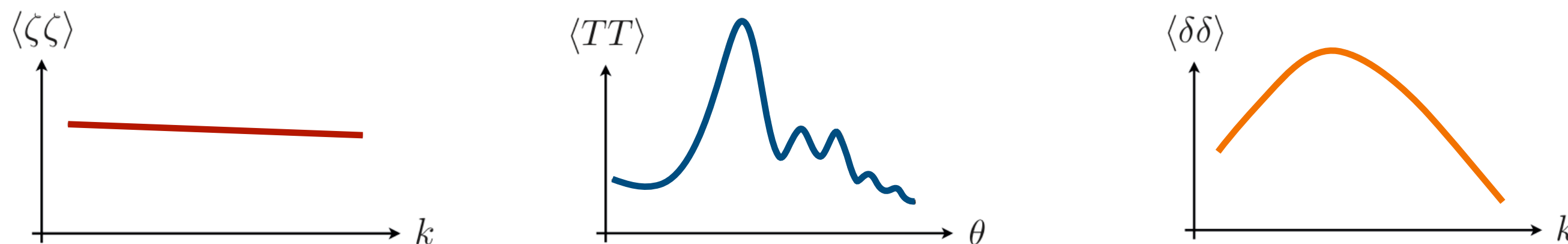
$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

Considering $\xi = \partial_t - H(t) x^i \partial_i$

$$\mathcal{L}_\xi g_{00} = \mathcal{L}_\xi g_{ij} = 0, \quad \mathcal{L}_\xi g_{0i} = -a^2 \dot{H} x^i$$

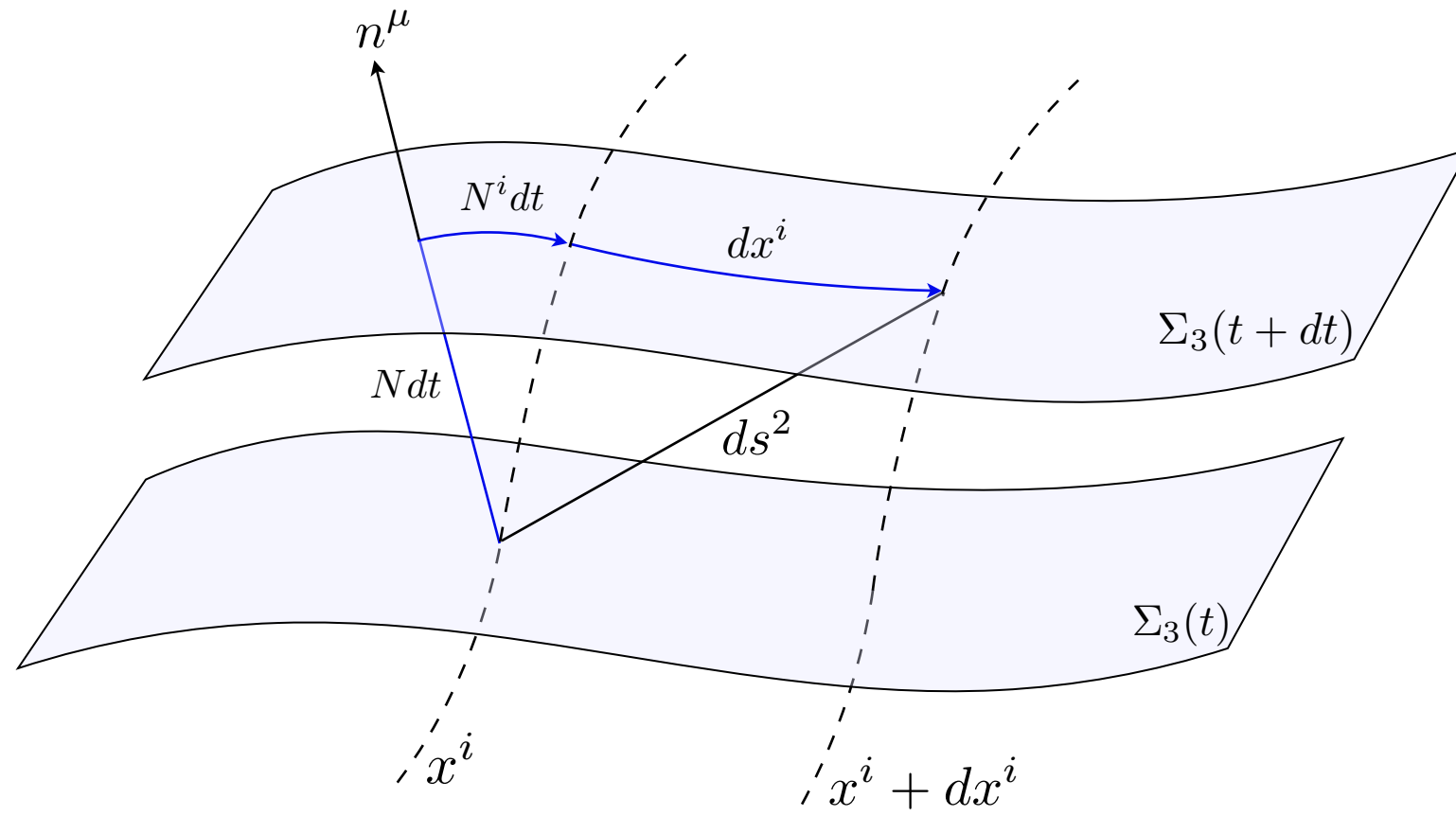
$\dot{H} \rightarrow$ order parameter

Goldstone theorem: it must exist a Goldstone boson $\pi(x)$



The EFT of inflation

Following the **bottom-up** approach, the most general EFT (in unitary gauge) compatible with unbroken spatial diffs $x^i \rightarrow x^i + \xi^i(t, \mathbf{x})$ can be constructed through the 3+1 decomposition of spacetime (ADM formalism) $\mathcal{M}_4 \rightarrow \mathbb{R} \times \Sigma_3$ [Cheung et. al (2007)]



The most general action is given by

$$S = \int d^4x \sqrt{-g} \mathcal{F}(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t)$$

Explicitly

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R + M_{\text{Pl}}^2 \dot{H} g^{00} - M_{\text{Pl}}^2 (3H^2 + \dot{H}) + \frac{M_2^4(t)}{2!} (\delta g^{00})^2 + \frac{M_3^4(t)}{3!} (\delta g^{00})^3 \right. \\ \left. - \frac{\bar{M}_1^3(t)}{2} (\delta g^{00}) \delta K^\mu{}_\mu - \frac{\bar{M}_2^2(t)}{2} (\delta K^\mu{}_\mu)^2 - \frac{\bar{M}_3^2(t)}{2} \delta K^\mu{}_\nu \delta K^\nu{}_\mu + \dots \right]$$

Time diff invariance can be restored through the Stückelberg procedure $t \rightarrow t + \pi(t, x)$

At energies $E \gg H$ the Goldstone mode and gravitational fluctuations decouple

$$S_\pi = \int d^4x \left[M_{\text{Pl}}^2 \dot{H} (\partial_\mu \pi)^2 + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{1}{a^2} (\partial_i \pi)^2 \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 - \frac{\bar{M}^2}{2} \frac{1}{a^4} (\partial_i^2 \pi)^2 + \dots \right]$$

Can we constrain the signs of $M_i, \bar{M}_i$ with positivity bounds?

- ~~Flat space~~ $\Rightarrow$ quasi dS
- ~~Scattering amplitudes~~ $\Rightarrow$ equal-time correlation functions
- ~~Lorentz invariance~~ $\Rightarrow$ spontaneously broken, bonus: no energy conservation

[Baumann et. al. (2015)]

[Grall & S. Melville (2021)]

[Creminelli et. al. (2023)]

[Freytsis et. al. (2023)]

[Green et. al. (2023)]

[Serra & Trombetta (2024)]

[Creminelli et. al. (2024)]

[Gordon Lee & S. Melville (2025)]

Lorentz breaking amplitudes
and
Amplitudes from dS correlators